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Date — 9/11/16

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Excellent

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Ans 1) Given $u = \tan^{-1} \frac{x^3+y^3}{x-y}$

$$\Rightarrow \tan u = \frac{x^3+y^3}{x-y}$$

$$\Rightarrow \tan u = \frac{x^3\left(1+\frac{y^3}{x^3}\right)}{x\left(1-\frac{y}{x}\right)}$$

$$\Rightarrow \tan u = \frac{x^2\left(1+\frac{y^3}{x^3}\right)}{\left(1-\frac{y}{x}\right)}$$

$$\Rightarrow \tan u = x^2 \phi$$

where $\tan u$ is a homogenous f^n of degree 2 and $\phi = \frac{\left(1+\frac{y^3}{x^3}\right)}{\left(1-\frac{y}{x}\right)}$

$\therefore$ By Euler's theorem.

$$x.\frac{\partial(\tan u)}{\partial x} + y.\frac{\partial(\tan u)}{\partial y} = 2\tan u$$

$$\Rightarrow x.\sec^2 u \frac{\partial u}{\partial x} + y.\sec^2 u \frac{\partial u}{\partial y} = 2\tan u$$

$$\Rightarrow \sec^2 u \left[x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right] = 2\tan u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\frac{\tan u}{\sec^2 u}$$

P.T.O

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{2\sin u}{\cancel{\cos u}} \cdot \cos^2 u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

Proved

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Ans 5) Given: $\log y = \tan^{-1} x$

(i) ~~diff. w.r.t. x~~ $\therefore y = e^{\tan^{-1} x}$

$$\cancel{\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{1+x^2}}$$ diff w.r.t x

$$y_1 = e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$\Rightarrow (1+x^2)y_1 = e^{\tan^{-1} x}$$

$$\Rightarrow (1+x^2)y_1 = y$$

again diff w.r.t x

$$\Rightarrow (1+x^2).y_2 + y_1.2x = y_1$$

$$\Rightarrow (1+x^2)y_2 + y_1 2x - y_1 = 0$$

$$\Rightarrow (1+x^2)y_2 + (2x-1)y_1 = 0 \quad -(1)$$

Proved

(ii) Now diff eq^n (1) n times by Leibnitz theorem

$$\left[(1+x^2)y_{n+2} + {}^nC_1 \cdot 2x \cdot y_{n+1} + {}^nC_2 \cdot 2 \cdot y_n\right] + (2x-1)y_{n+1} + {}^nC_1 \cdot 2 \cdot y_n = 0$$

$$\Rightarrow (1+x^2)y_{n+2} + 2xn \cdot y_{n+1} + n(n-1)\cdot y_n + (2x-1)y_{n+1} + 2n y_n = 0$$

$$\Rightarrow (1+x^2)y_{n+2} + (2xn+2x-1)y_{n+1} + n(n+1)y_n = 0$$

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Proved

Ans 4) Given! $y = \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

$$\Rightarrow y\sqrt{1-x^2} = \sin^{-1}x$$

Squaring both sides

$$\Rightarrow y^2(1-x^2) = (\sin^{-1}x)^2$$

Now diff w.r.t x

$$\Rightarrow (1-x^2).2y.y_1 + y^2(-2x) = 2\sin^{-1}x.\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)2y.y_1 - 2xy^2 = \cancel{2\sin^{-1}x}\ 2y$$

$$\Rightarrow 2y\left[(1-x^2)y_1 - xy\right] = 2y$$

$$\Rightarrow (1-x^2)y_1 - xy = 1$$

again diff wrt x.

$$\Rightarrow y_1(-2x) + (1-x^2)y_2 - xy_1 - y.1 = 0$$

$$\Rightarrow -2xy_1 + (1-x^2)y_2 - xy_1 - y = 0$$

$$\Rightarrow (1-x^2)y_2 - 3xy_1 - y = 0$$

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Proved

Ans 3) Given $y = \cos(m \sin^{-1} x)$

diff. w.r.t x.

$\Rightarrow y_1 = -\sin(m \sin^{-1} x) \cdot \frac{m}{\sqrt{1-x^2}}$

$\Rightarrow \sqrt{1-x^2}\, y_1 = -m \sin(m \sin^{-1} x)$

Sq both sides

$\Rightarrow (1-x^2) y_1^2 = m^2 \sin^2(m \sin^{-1} x)$

$\Rightarrow (1-x^2) y_1^2 = m^2 [1 - \cos^2(m \sin^{-1} x)]$

$\Rightarrow (1-x^2) y_1^2 = m^2 [1 - y^2]$

again diff w.r.t x.

$\Rightarrow (1-x^2) \cdot 2y_1 y_2 + y_1^2(-2x) = m^2(-2y \cdot y_1)$

$\Rightarrow (1-x^2) \cdot 2y_1 \cdot y_2 - 2x y_1^2 = -m^2 2y \cdot y_1$

$\Rightarrow \cancel{2y_1}[(1-x^2) \cdot y_2 - x y_1] = -m^2 \cancel{2} y \cdot \cancel{y_1}$

$\Rightarrow (1-x^2) y_2 - x y_1 + m^2 y = 0$

Proved

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Ans 2) Given: $y = \cos(10 \cos^{-1} x)$

diff wrt x

$y_1 = -\sin(10\cos^{-1} x) \cdot \frac{-10}{\sqrt{1-x^2}}$

$\Rightarrow (\sqrt{1-x^2})\, y_1 = 10 \cdot \sin(10\cos^{-1} x)$.

Sq both sides.

$\Rightarrow (1-x^2)\, y_1^2 = 100 \cdot \sin^2(10\cos^{-1} x)$

$\Rightarrow (1-x^2)\, y_1^2 = 100[1 - \cos^2(10\cos^{-1} x)]$

$\Rightarrow (1-x^2)\, y_1^2 = 100[1 - y^2]$

again diff w.r.t x.

$\Rightarrow (1-x^2) \cdot 2y_1 \cdot y_2 + y_1^2(-2x) = 100(-2y \cdot y_1)$

$\Rightarrow (1-x^2)\, 2y_1 \cdot y_2 - 2x y_1^2 = -200\, y \cdot y_1$

$\Rightarrow \cancel{2y_1}[(1-x^2) \cdot y_2 - x y_1] = -\cancel{200}^{100}\, y \cdot \cancel{y_1}$

$\Rightarrow (1-x^2)\, y_2 - x y_1 + 100 y = 0 \quad -(1)$

~~again diff wrt x.~~

$\cancel{(1-x^2) y_3 + y_2(-2x) - x y_2 + y_1 \cdot 1 + 100 y_1 = 0}$

$\cancel{\Rightarrow (1-x^2) y_3 - 2x y_2 - x y_2 + y_1 + 100 y_1 = 0}$

$\cancel{\Rightarrow (1-x^2) y_3 - 3x y_2 + 101 y_1 = 0}$

Diff eqⁿ (1) n times by Leibnitz theorem.

$(1-x^2)y_{n+2} + {}^nC_1(-2x)\cdot y_{n+1} + {}^nC_2(-2)\cdot y_n$
$- xy_{n+1} + {}^nC_1\cdot 1\cdot y_n + 100\cdot y_n = 0$

$\Rightarrow (1-x^2)y_{n+2} - 2nx\cdot y_{n+1} - n(n-1)\cdot y_n - xy_{n+1}$
$- ny_n + 100y_n = 0$

$\Rightarrow (1-x^2)y_{n+2} - xy_{n+1}[2n+1] - ny_n[n-1+1] + 100y_n = 0$

$\Rightarrow (1-x^2)y_{n+2} - xy_{n+1}[2n+1] - n^2y_n + 100y_n = 0$

$\Rightarrow (1-x^2)y_{n+2} - xy_{n+1}[2n+1] - y_n[n^2-100] = 0.$

Putting $n = 10$

$\Rightarrow (1-x^2)y_{12} - xy_{11}[2\times 10+1] - y_{10}[10^2-100] = 0$

$\Rightarrow (1-x^2)y_{12} - 21xy_{11} = 0$

$\Rightarrow (1-x^2)y_{12} = 21xy_{11}$

Proved

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